A TRANSISTOR SAWTOOTH GENERATOR

R. R. DUTTA GUPTA

INSTITUTE OF RADIOPHYSICS AND ELECTRONICS,
UNIVERSITY COLLEGE OF TECHNOLOGY,
CALCUTTA 9.

(*Received August*, 24 1963)

ABSTRACT. Principal methods employed in improving the linearity of the output of a sawtooth generator are briefly described. The non-linearity caused by loading is also critically examined. A technique developed to compensate for the non-linearity is then presented. It is shown that the technique also enables one to obtain a reverse curvature which may be of importance in case the sawtooth output needs to be amplified before it can be of practical use.

INTRODUCTION

Sawtooth waveforms having a high degree of linearity are required for a variety of purposes in electronic circuit applications. In current practice the Miller Integrator and the Bootstrap circuit have become standard methods for the production of such a linear waveform. (Chance *et al.*, 1949 and Bedford and Stevens, B.P. 474623). Neither of these devices can however either preserve a good linearity of waveform in the presence of loading or give a waveform having a reverse curvature. In the present paper we shall describe a fully transistorized arrangement that not only gives a linearity approaching that obtainable with any of the above circuits but is also capable of preserving the linearity even in the presence of resistive loading. An additional feature of this arrangement is that it permits one to obtain a wave form having a reverse curvature that can be adjusted smoothly to any assigned degree.

The paper starts with a brief reference to the existing arrangements for obtaining good linearity and their basic limitations. The theory of operation of the proposed new arrangement is then outlined. Results obtained with a practical circuit based on the theory are given and discussed briefly in the concluding Section.

A BRIEF RESUME OF THE EXISTING METHODS OF IMPROVING LINEARITY

Normally, pentode charging gives a reasonably good linear waveform for all practical purposes. For precision work requiring a much higher degree of linearity, use is generally made of any one of the following :

(i) inverse curvature of a valve characteristic,
(ii) auxiliary time constant circuit and
(iii) feedback.

A typical example using inverse curvature of a valve characteristic is the Bedford and Stevens' method of linearization. In this method the condenser is charged through a resistance to obtain an exponential waveform first. A portion of this is then fed at the input of an amplifier whose characteristic is shaped and the input and anode load so proportioned that at each point on the dynamic characteristic, the anode potential is equal to the difference between the p.d. across the condenser and the potential required to produce a linear characteristic. Obviously, then the resultant potential difference between the condenser and the anode of the amplifier gives a sweep that is linear.

A circuit involving the use of an auxiliary time constant is due to Hawkins (B.P. 511600) and is shown in Fig. 1. In this the product C_3R_2 is made very large

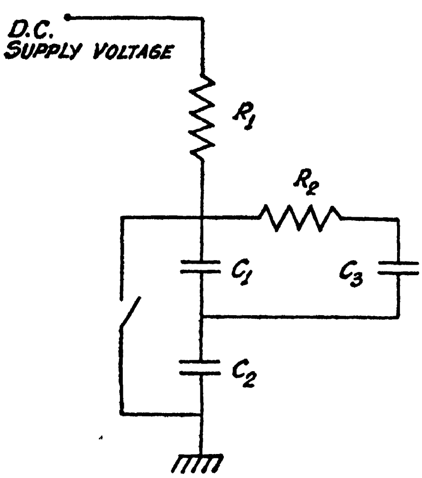


Fig. 1. Sawtooth generator with auxiliary time constant circuit.

so that when the condensers C_1 and C_2 are charged in series, C_3 acquires a voltage that is smaller than that across C_1. Again, when C_1 and C_2 are completely discharged by the switch, C_3 still retains a charge owing to the large time constant C_3R_2. Therefore, at the beginning of the charging period, C_1 is charged both from the supply voltage and from C_3. This continues till the potential across C_3 stops drooping and begins to rise again due to increased potential across C_1. The actual nature of voltage waveform across C_3 is thus approximately parabolic while that across C_1 and C_2 exponential. The two may be made to cancel each other's curvature so that upon taking the sum one gets a waveform which is linear to a very high degree.

A highly linear sawtooth generator utilising feedback is due to Blumlein (BP. 400976) and is called the Miller Integrator after the name of J. M. Miller who first observed the effect of grid-anode capacitance on the input impedance of a valve amplifier (Miller). The integrator is shown schematically in Fig. 2. If the amplifier be supposed to have a voltage gain of $-A$ from grid to anode, the effective capacitance between grid and cathode becomes $(1+A)C$ where C is the capacitance between grid and anode. Thus, the integrating circuit at the input, consisting of R and $(1+A)C$, effectively becomes a circuit of very large time cons-

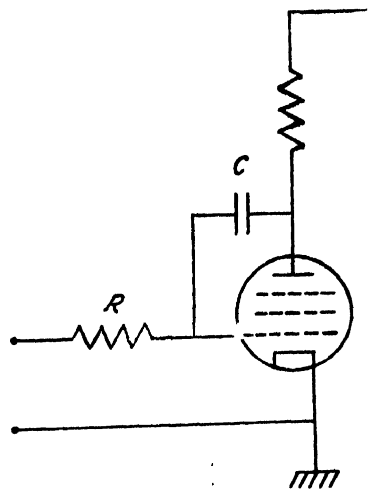


Fig. 2. Miller Integrator.

tant. As such, with a square wave drive the voltage swing obtainable at the grid of the amplifier is a very small fraction of the total supply voltage and hence the output waveform is extremely linear.

Another form of a precision sawtooth generator using feedback, known as Bootstrap Circuit, is shown in Fig. 3. The operation of the circuit may be des-

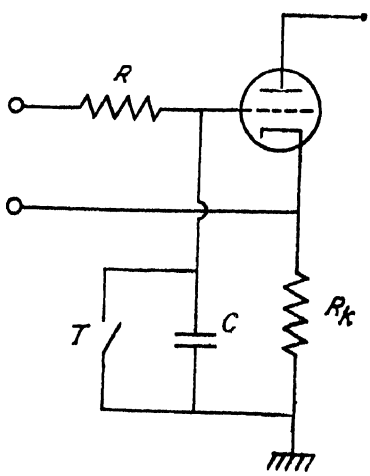


Fig. 3. Bootstrap circuit.

cribed as follows. If the cathode follower be supposed to possess unity gain, the voltage appearing across C also appears across R_k undiminished and in the same polarity so that the voltage across the resistance R remains constant. The current through R therefore remains constant, charging the condenser C linearly with time until it is discharged by the switch T.

Considering both the Miller and the Bootstrap Circuits, it is implicit that the loading across the condensers must be completely absent as otherwise the exponential curvature would tend to appear. Neither of these is capable of giving a waveform having a reverse curvature. As such with these it is not possible, by any simple means to compensate for the exponential response of the amplifier stages which might have to be used subsequently under certain circumstances.

THE PROPOSED LINEARISING CIRCUIT AND ITS THEORY OF OPERATION

Current and voltage in a shunt RC network :

The relevance of examining the action of current through a parallel combination of R and C is obvious from the preceding section. Denoting current and voltage as functions of time t, viz. $i(t)$ and $v(t)$ respectively as shown in Fig. 4, the relation between them is found to be given by

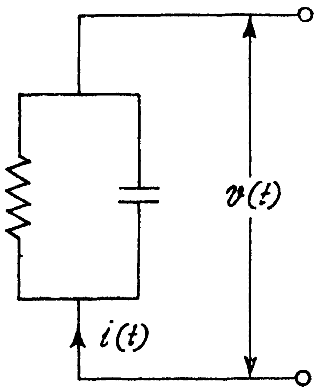


Fig. 4. Shunt RC network.

$$v(t) = Ke^{-\frac{t}{RC}} + e^{-\frac{t}{RC}} \int e^{\frac{t}{RC}} \cdot \frac{i(t)}{C} \cdot dt \tag{1}$$

where K is a constant of integration to be fixed by initial conditions.

It is easy to see from eqn. (1) that even if the charging current (it) be kept constant the voltage waveform across the combination of Fig. 4 does not increase linearly with time. As such, in order to obtain a linear sawtooth voltage across the combination, it is necessary that the current $i(t)$ should vary in such a manner that $v(t)$ in eqn. (1) becomes linearly related with time. We discuss below a practical arrangement for obtaining the required functional dependence of $i(t)$.

Schematic description of the circuit :

The suggested method for linearising the waveform through charging a shunt R–C combination is shown schematically in Fig. 5. This utilises a pair of transistors, one $n-p-n$ and the other $p-n-p$. C and R in parallel with the admittance arising out of the loading effect of T_2 form the circuit

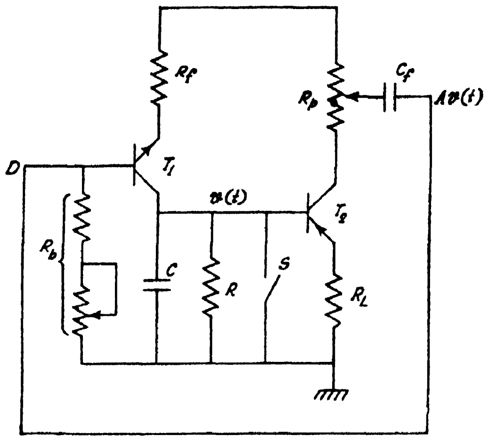


Fig. 5. Schematic arrangement for Transistor Sawtooth Generator.

being charged by T_1. In the absence of the feedback link marked D, T_1 delivers a constant charging current. As was mentioned before, this does not produce a linear waveform across C. With the feedback link, however, T_2 acts as an amplifier and provides a feedback current to T_1 which is dependant upon time in such a manner that the voltage across C becomes a linear function of time. As is evident, the arrangement under consideration belongs to the same family as that of Miller Integrator or Bootstrap Circuit. In contrast to these latter devices, however, this utilises a feedback, the magnitude of which can be controlled smoothly over a wide range. It is then possible with proper increase in feedback to even turn round the waveform giving a curvature in the reverse direction.

Analysis of the linearising circuit :

We adopt the following nota- tion in the analysis.

β_1 : grounded-emitter current gain of T_1.

R_b : resistance in the base circuit of T_1 controlling the collector bias current.

C : capacitance to be charged.

R : load resistance across C.

β_2 : grounded-emitter current gain of T_2.

R_L : resistance in the emitter circuit of T_2.

R_p : potentiometer on the collector of T_2 giving variable feedback to T_1.

$v(t)$: instantaneous voltage across C.

A : effective voltage amplification of T_2 from base to the variable point of R_p.

On the basis of the plausible assumption that the loading effect of R_f and R_b on the amplification factor A is negligible, the collector current of T_1 may be written as given by $Av(t)/R_f$, $-Av(t)$ being the voltage feedback to T_1 from the output of T_2. Thus at any instant t, the total collector current in T_1 is given by

$$i(t) = I_0+\frac{Av(t)}{R_f}. \qquad \dots (2)$$

where I_0 is the quiescent current in the collector of T_1 and this is the net current charging the combination of the condenser C and the effective resistance across it, viz, R and $\beta_2 R_L$ in parallel. Denoting this effective resistance by R' and replacing R by R' in eqn. (1) we write,

$$v(t) = Ke^{-\frac{t}{R'C}}+e^{-\frac{t}{R'C}}\int e^{\frac{t}{R'C}}\frac{\left[I_0+\frac{Av(t)}{R_f}\right]}{C}dt.$$

or,
$$v(t) = Ke^{-\frac{t}{R'C}}+I_0R'+\frac{A}{R_fC}\cdot e^{-\frac{t}{R'C}}\int e^{\frac{t}{R'C}}v(t)dt. \qquad \dots (3)$$

It would be convenient to consider the solution of eqn.(3) under two separate heads viz (a) solution for the special case when we have an ideal sawtooth waveform and (*b*) the general solution.

Condition for generation of sawtooth waveform :

For $v(t)$ in eqn. (3) to be linear, $v(t) = \theta t$, where θ is a constant determining the slope of the voltage swing. Thus eqn(3) becomes,

$$\theta t = Ke^{-\frac{t}{R'C}}+I_0R'+\frac{A}{R_fC}e^{-\frac{t}{R'C}}\int\cdot v(t)\cdot dt. \qquad \dots (4)$$

It can be shown from eqn. (4) (see Appendix) that the conditions for generation of sawtooth waveform is given by.

$$\frac{AR'}{R_f} = 1, \qquad \dots (5)$$

and
$$I_0 = \frac{A\theta R'C}{R_f} \qquad \dots (6)$$

Inserting (5) in (6), we get,

$$I_0 = \theta C \qquad \dots (7)$$

Eqn. (7) reveals that the slope of the linear sweep is the same as would be obtained due to the charging of C by a constant current I_0 only. The feedback voltage at D appears just to have compensated for the loading across C.

General solution :

In order to obtain the general solution of eqn. (3) we put

$$\tau = R'C, \qquad \dots (8)$$

and get upon differentiation

$$\frac{dv(t)}{dt}+v(t)\left(\frac{1}{\tau}-\frac{A}{R_fC}\right)=\frac{I_0}{C}. \qquad \dots (9)$$

The solution of this equation is obtained as

$$v(t) = Be^{-\left(\frac{1}{\tau} - \frac{A}{R_fC}\right)t} + e^{-\left(\frac{1}{\tau} - \frac{A}{R_fC}\right)t} \int e^{\left(\frac{1}{\tau} - \frac{A}{R_fC}\right)t} \cdot \frac{I_0}{C} \cdot dt$$

or,
$$v(t) = Be^{-\left(\frac{1}{\tau} - \frac{A}{R_fC}\right)t} + \frac{I_0}{C\left(\frac{1}{\tau} - \frac{A}{R_fC}\right)}, \qquad \ldots (10)$$

where B is a constant of integration. As $v(t) = 0$ at $t = 0$, eqn. (10) becomes,

$$v(t) = \frac{I_0}{C\left(\frac{1}{\tau} - \frac{A}{R_fC}\right)} \left[1 - e^{-\left(\frac{1}{\tau} - \frac{A}{R_fC}\right)t}\right]. \qquad \ldots (11)$$

Using (8), eqn. (11) becomes,

$$v(t) = \frac{I_0}{\left(\frac{1}{R'} - \frac{A}{R_f}\right)} \left[1 - e^{-\left(\frac{1}{R'} - \frac{A}{R_f}\right)\frac{t}{C}}\right], \qquad \ldots (12)$$

which is the general solution required. The constant $\left(\frac{1}{R'} - \frac{A}{R_f}\right)$ may be adjusted to have a value positive, negative or approaching zero. This enables generation of waveform as mentioned below.

Case I : Undercompensated :

When

$$\frac{1}{R'} >> \frac{A}{R_f}, \qquad \ldots (13)$$

eqn(12) gives an exponential waveform with a slope falling with time, the maximum value being given by $\frac{I_0}{\left(\frac{1}{R'} - \frac{A}{R_f}\right)}$ for t approaching infinity.

Case II. Critically compensated :

In the limiting case when

$$\frac{1}{R'} \rightarrow \frac{A}{R_f}, \qquad \ldots (14)$$

eqn. (12) may be expanded retaining only the linear term in t. This gives

$$v(t) = \frac{I_0}{C} \cdot t, \qquad \ldots \quad (15)$$

ensuring a linear waveform having a constant slope equal to $\frac{I_0}{C}$ as deduced before in eqn. (7).

Case III. Overcompensated :

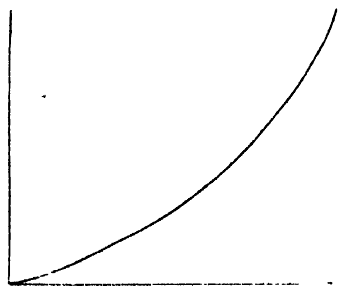
When

$$\frac{1}{R'} < \frac{A}{R_f}, \qquad \ldots \quad (16)$$

eqn. (12) on being differentiated twice gives,

$$\frac{d^2v(t)}{dt^2} = + \frac{I_0}{C}\left(\frac{A}{R_f} - \frac{1}{R'}\right)e^{+\left(\frac{A}{R_f} - \frac{1}{R'}\right) \cdot \frac{t}{C}} \qquad \ldots \quad (17)$$

Inserting the condition of (16) in eqn. (17) we see that $\frac{d^2v(t)}{dt^2}$ is positive implying thereby that the slope increases with time as shown in Fig. 6. The



$t \rightarrow$

Fig. 6. Increasing slope with time.

amplitude of the waveform will be limited by the time interval in between switching across the condenser, i.e., by the switching frequency.

We now proceed to describe some experimental results in support of the relations deduced above.

EXPERIMENTAL RESULTS

The complete circuit diagram of the experimental set up is shown in Fig. 7 where T_1 (OC139) is the charging transistor, T_2 (OC72), the amplifier and T_3 (OC44) the blocking oscillator for switching the condenser C, the waveform being observed across it. The experimental results demonstrating the validity of the relationship derived earlier for the linear case viz, eqns. (5) and (7) are now outlined first.

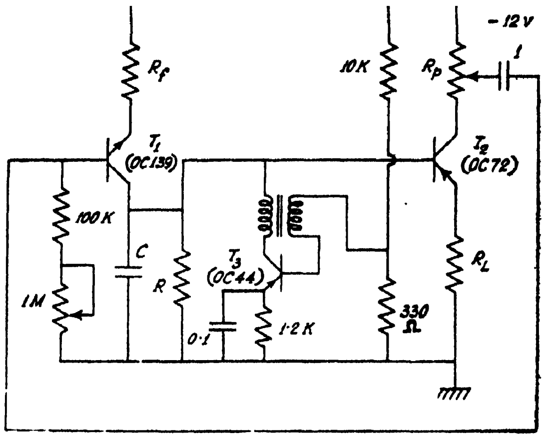


Fig. 7. Experimental arrangement for the Transistor Sawtooth Generator.

Table I gives the observed slope as a function of the capacity C for $I_0 = 0.32$ ma. as well as the theoretical values of the slope as given by the equation,

$$I_0 = 0C. \qquad \text{... (7)}$$

TABLE I

Charging current I_0(ma)	Capacitance C (μf)	Experimental slope (mv/μsec)	Theoretical slope (mv/μsec)
0.32	0.05	6.2	6.4
	0.10	3.2	3.2
	0.20	1.5	1.6
	0.52	0.6	0.6
	1.00	0.3	0.32

Columns 3 and 4 reveal satisfactory agreement between the experimental and calculated values of the slope.

Table II gives the observed slope as a function of the current I_0 for $C = 0.1\mu f$ as well as the theoretical value obtained from eqn. (7). In this case also the agreement between the experimental and calculated values is found to be excellent.

In order to seek experimental support for eqn. (5) it is necessary to express R' and A in explicit form relevant to the circuit configuration as shown in Fig. 7. The effective resistance R' in this latter case is a shunt combination of R and $\beta_2 R_L$ and is given by

$$R' = \frac{\beta_2 R_L R.}{(\beta_2 R_L + R)}. \qquad \text{... (18)}$$

TABLE II

Capacitance $C(\mu f)$	Charging current I_0(ma)	Experimental slope ($mv/\mu sec$)	Theoretical slope ($mv/\mu sec$)
0.1	0.67	6.4	6.7
	0.58	5.8	5.8
	0.50	5.0	5.0
	0.42	4.4	4.2
	0.33	3.3	3.3
	0.25	2.4	2.5

Also from Fig. 7, since the currents in the emitter and collector circuits of T_2 are nearly equal and so also the emitter and base voltages, the gain at the collector of T_2 would be given by the ratio of the collector to emitter resistances, $\frac{R_p}{R_L}$. If R_p be tapped for a fraction γ of the total output voltage,the effective gain A would be given by,

$$A = \frac{R_p}{R_L} \cdot \gamma. \qquad \dots (19)$$

Using (5), (18) and (19) and rearranging one gets,

$$R_p\gamma = \frac{R_f(R+\beta_2 R_L)}{\beta_2 R}. \qquad \dots (20)$$

Eqn. (20) shows that for a fixed value of R_f, R, R_L and β_2, the product $R_p\gamma$ should be a constant for linear waveform. Table III confirms this result for $R_f = 600$ ohms, $R = 30K\Omega$, $R_L = 50$ ohms and $\beta_2 = 100$. Columns 3 and 4 show satisfactory agreement between the experimental and calculated values of $R_p\gamma$.

Referring to Fig. 7 and rearranging eqn. (20)

$$\frac{\gamma}{R_f} = \frac{R+\beta_2 R_L}{\beta_2 R\, R_p}. \qquad .. (20a)$$

TABLE III

R_p (ohms)	γ	$R_p\gamma$ (experimental)	$R_p\gamma$ (calculated)
500	0.013	6.50	7.00
325	0.021	6.82	
260	0.026	6.80	
180	0.037	6.66	
135	0.048	6.50	

Columns 3 and 4, Table IV again show agreement between the experimental and calculated values of $\frac{\gamma}{R_f}$ based on (20a) with the following values of the various parameters :

$$R = 30\text{K}\Omega,\ \beta_2 = 100,\ R_L = 50 \text{ ohms and } R_L = 325 \text{ ohms.}$$

TABLE IV

R_f (ohms)	γ	γ/R_f (experimental)	γ/R_f (calculated)
130	0.004	0.31×10^{-4}	0.35×10^{-4}
300	0.010	0.33×10^{-4}	
600	0.018	0.30×10^{-4}	
1200	0.037	0.31×10^{-4}	
2400	0.081	0.34×10^{-4}	
4700	0.155	0.33×10^{-4}	

TABLE V

R_L (ohms)	R ($K\Omega$)	$\gamma\times10^3$ (experimental)	$\gamma\times10^3$ (calculated)
50	30	21	22
	14.2	25	25
	10	27	28
	7.5	28	30
	4.3	37	40

TABLE VI

R (k Ω)	R_L (ohms)	$\gamma\times10^3$ (experimental)	$\gamma\times10^3$ (calculated)
30	10	19	18.6
	30	20	19.6
	50	21	21.0
	100	25	24.0
	200	30	30.0
	300	35	36.0

Tables V and VI show the experimental and calculated values of γ for various values of R and R_L respectively. The agreements on the basis of calculations, according to eqn. (20) are again seen to be good.

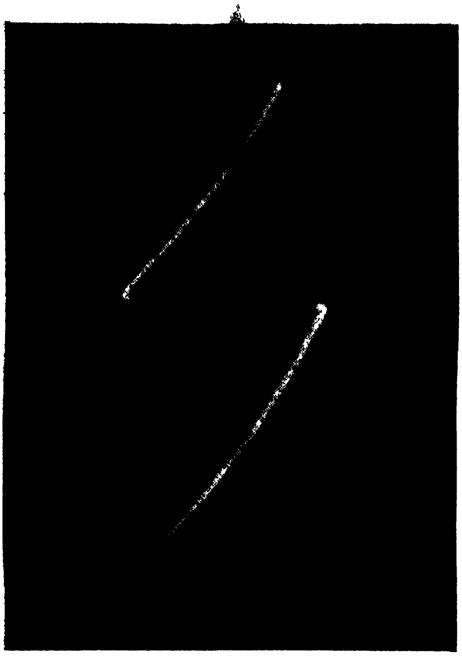


Fig. 8. Undercompensation.

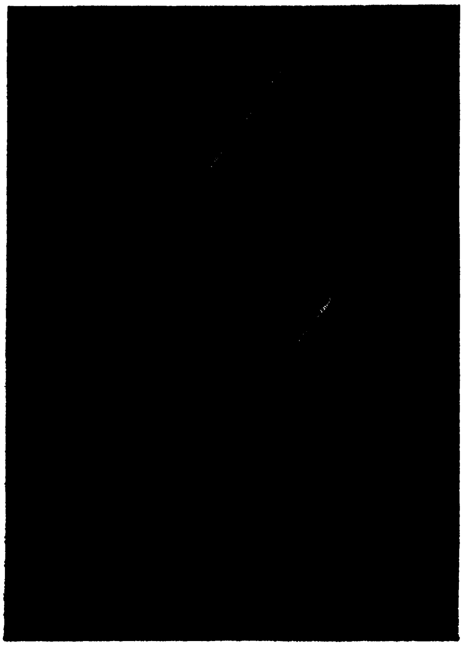


Fig. 9. Critical compensation.

Fig. 10. Overcompensation.

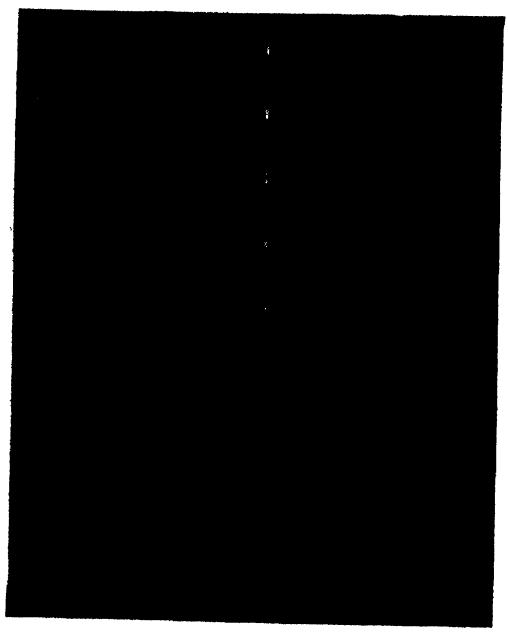


Fig. 11. Linear sawtooth with markers.

It has been shown earlier that three cases corresponding to under, critical and overcompensation are possible. The photographs representing them are presented in Figs. 8, 9 and 10 respectively. The linear sweep of Fig. 9 is also displayed in Fig. 11 with markers in order to illustrate its linearity.

DISCUSSIONS

It has been seen with the proposed circuit that three possible types of waveform may be generated. Though surperfluous at first sight, the overcompensated case may be of considerable importance in linearising the ultimate waveform. For even supposing that a perfectly linear waveform is generated, it often needs amplification before adequate amplitude is obtained.

If the amplifier includes a coupling network, as shown in Fig. 12, then for a linear current driving through the network,

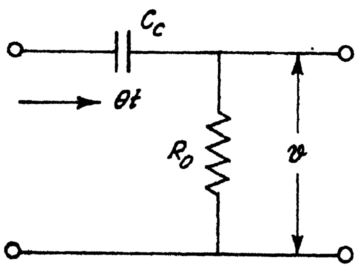


Fig. 12. Coupling network.

$$\partial(p) = \frac{\theta}{p^2} \cdot \frac{pC_cR_0}{(C+pC_cR_0)}, \quad \ldots \quad (21)$$

in transform notations where θt has been assumed to be the driving current whose transform is $\frac{\theta}{p^2}$. Taking the inverse transform of (21),

$$v(t) = \theta R_0 C_c \left(1 - e^{-\frac{t}{R_0 C_c}} \right). \quad \ldots \quad (22)$$

Thus (22) shows that even for a strictly linear drive the output voltage becomes exponential. Therefore if the drive θt is given an initial reverse curvature (Fig. 10), the amplifier output will tend to compensate for this and deliver an output waveform having reasonably good linearity.

ACKNOWLEDGMENT

The author intends to thank Prof. J. N. Bhar for his kind interest in the work. He is particularly indebted to Prof. S. Deb for constant advice and helpful discussions throughout the course of the work. Thanks are also extended to the Council of Scientific and Industrial Research for appointing him as Pool Officer.

REFERENCES

Starr, A. T., 1953, Radio and Radar Technique, Sir Isaac Pitman & Sons, Ltd., London, p. 491.

Chance B., (*et. al*), 1949, Waveforms, Radiation laboratory Series, No. 19, McGraw-Hil Book Co., Inc., p. 31-37.

Bedford, L. H. and Stevens, W. H., British Patent 474623.

Hawkins, G. F., British Patent 511600.

Blumlein, A. D., British Patent 400976.

Miller, J. M., Dependence of the Input Impedance of a 3-element Vacuum Tube upon the Load in the Plate Circuit, Nat. Bur. Standards Sci., Paper 351.

APPENDIX

We rewrite eqn. (4) from the text for the sake of convenience.

$$\theta t = Ke^{-\frac{t}{R'C}} + I_0R' + \frac{A}{R_fC} e^{-\frac{t}{R'C}} \int e^{\frac{t}{R'C}} v(t)dt. \tag{1}$$

Noting that,

$$\int e^{\frac{t}{R'C}} . \theta t \cdot dt = (\theta tR'C - \theta R'^2C^2) e^{\frac{t}{R'C}}$$

eqn. (1) becomes,

$$\therefore \quad \theta t = Ke^{-\frac{t}{R'C}} + I_0R' + \frac{A}{R_fC}(\theta tR'C - \theta R'^2C^2). \tag{2}$$

Putting $t = 0$ in eqn. (2) one gets,

$$K = -\left(I_0R' - \frac{A}{R_fC} \cdot \theta . R'^2C^2).\right). \tag{3}$$

From (2) and (3),

$$\theta t = \theta \cdot \frac{AR'}{R_f} . t + R'\left(I_0 - \frac{A\theta R'C}{R_f}\right)(1 - e^{-\frac{t}{R'C}}). \tag{4}$$

Thus identity (4) gives the conditions for linear sweep as

$$\frac{AR'}{R_f} = 1, \tag{5}$$

and

$$I_0 = \frac{A\theta R'C}{R_f}. \tag{6}$$